Unit 4: Regression and Prediction

# 3. Inference for Linear Regression

(Chapter 5.4)

3/28/2022

# Recap from last time

1. We can use the slope and intercept of a regression line to make predictions
2. We can also sometimes extrapolate, but this can be fraught
3. Like other statistics we've explored so far, linear regression models are appropriate only when some conditions are met

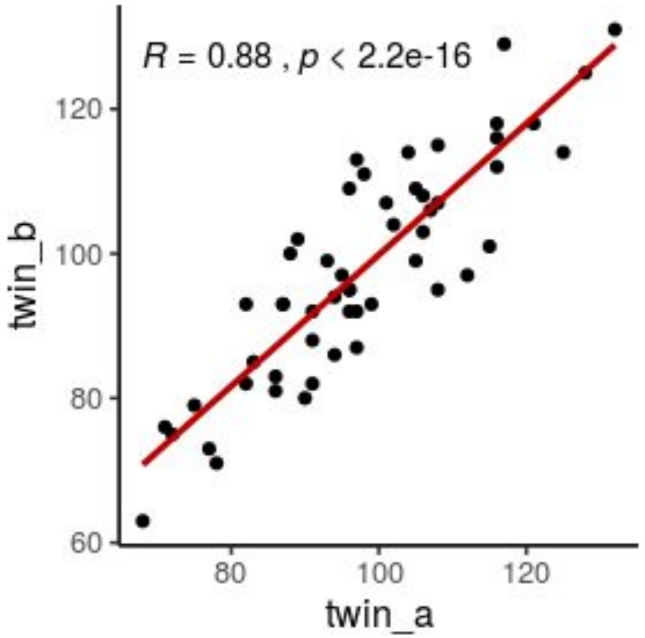
# Key ideas

1. A regression model's slope codes the relationship between the two measures
2. Correlation is equivalent to the slope of a regression for standardized values
3. Inference for regression parameters uses t-tests

# Nature or nurture?

In 1966 Cyril Burt published a paper called "**The genetic determination of differences in intelligence: A study of monozygotic twins reared apart**"

The data consist of IQ scores for [an assumed random sample of] 53 identical twins, separated within 6-months of birth and raised apart



# Practice Question 1: Interpreting regression output

```
              Estimate   Std. Error t value Pr(>|t|)
(Intercept)  9.08670    6.92036    1.313    0.195
twin_a       0.90741    0.07004   12.957   <2e-16 ***
---
Residual standard error: 7.417 on 51 degrees of freedom
Multiple R-squared:  0.767,    Adjusted R-squared:  0.7624
F-statistic: 167.9 on 1 and 51 DF,  p-value: < 2.2e-16
```

**Which of the following is <u>false</u>?**

(a) An additional 10 points in one twin's IQ is associated with additional 9 points in the the other twin’s IQ, on average.

(b) Roughly 91% of the variance in twins' IQs can be predicted by the model.

(c) The linear model is `twin_b = 9.08 + 0.91 x twin_a`.

(d) Twins in group b with IQs higher than average IQs tend to have biological twins in group a with higher than average IQs as well.

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# Practice Question 2: Testing the relationship

Assuming that these 53 pairs of twins are a representative sample of all twins separated at birth, we would like to test if these data provide convincing evidence that the IQ of a biological twin is a significant predictor of IQ of the other twin.

**What are the appropriate hypotheses?**

$\hat{y} = \beta_0 + \beta_1 x$

(a) $H_0: b_0 = 0$; $H_A: b_0 \neq 0$

(b) $H_0: \beta_0 = 0$; $H_A: \beta_0 \neq 0$

(c) $H_0: b_1 = 0$; $H_A: b_1 \neq 0$

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# Analyzing the slope of the regression line

| | estimate | std.error | t-value | p-value |
|---|---|---|---|---|
| (Intercept) | 9.0867 | 6.9203 | 1.3130 | 0.1950 |
| twin_a | 0.9074 | 0.0700 | 12.956 | 0.0000 |

We always use a **t-test** in inference for regression.

Remember: test statistic *T = (point estimate - null value) / SE*

Point estimate: $b_1$ is the observed slope. $SE_{b1}$ is the standard error of the slope.

Degrees of freedom of the slope is *df = n - 2*, where n is the sample size.

Remember: we lose 1 degree of freedom for each parameter we estimate, and in simple linear regression we estimate 2 parameters, $\beta_0$ and $\beta_1$.

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$$T = \frac{.9074 - 0}{.0700} = 12.956$$

$$df = 53 - 2 = 51$$

$$p - value = P(|T| > 12.956) < .001$$

# What is the relationship between slope and correlation?

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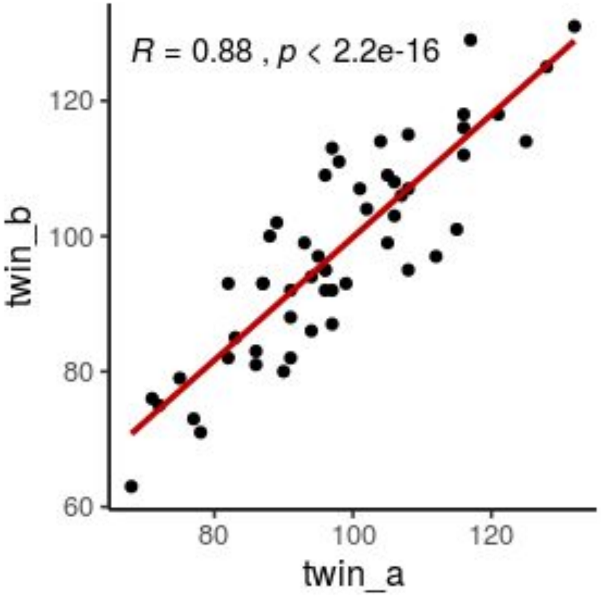

# Practice Question 3: Confidence intervals for regression estimates

Remember that a confidence interval is calculated as point estimate ± ME and the degrees of freedom associated with the slope in a simple linear regression is n - 2. **Which of the below is the correct 95% confidence interval for the slope parameter? (Note that the model is based on observations from 53 twins).**

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(a) 9.0867 ± 1.65 x 6.9203

(b) .9074 ± 2.01 x .0700

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**(b) .9074 ± 2.01 x .0700**

(c) .9074 ± 1.96 x .0700

(d) 9.0867 ± 1.96 x .0700

*$n = 53$ $df = 53 - 2 = 51$*

*95%: $t_{51}^* = 2.01$*

*0.9074 ± 2.01 x 0.0700*

*(0.767, 1.05)*

# Inference for linear regression

Inference for the slope for a single-predictor linear regression model:

Hypothesis test: $T = \frac{b_1 - null\ value}{SE_{b_1}}$ $\quad df = n - 2$

Confidence interval: $b_1 \pm t^{\star}_{df=n-2} SE_{b_1}$

The null value is often 0 since we are usually checking for **any** relationship between the explanatory and the response variable.

The regression output gives $b_1$, $SE_{b1}$, and **two-tailed** p-value for the t-test for the slope where the null value is 0.

We rarely do inference on the intercept, so we'll focusing on the slope.

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