

# Unit 4: Regression and Prediction

## 3. Inference for Linear Regression (Chapter 5.4)

11/26/2018

# Quiz 8 - linear regression

# Recap from last time

1. We can use the slope and intercept of a regression line to make predictions
2. We can also sometimes extrapolate, but this can be fraught
3. Just like t-tests and the other statistics we've explored so far, linear regression models are appropriate only when some conditions are met

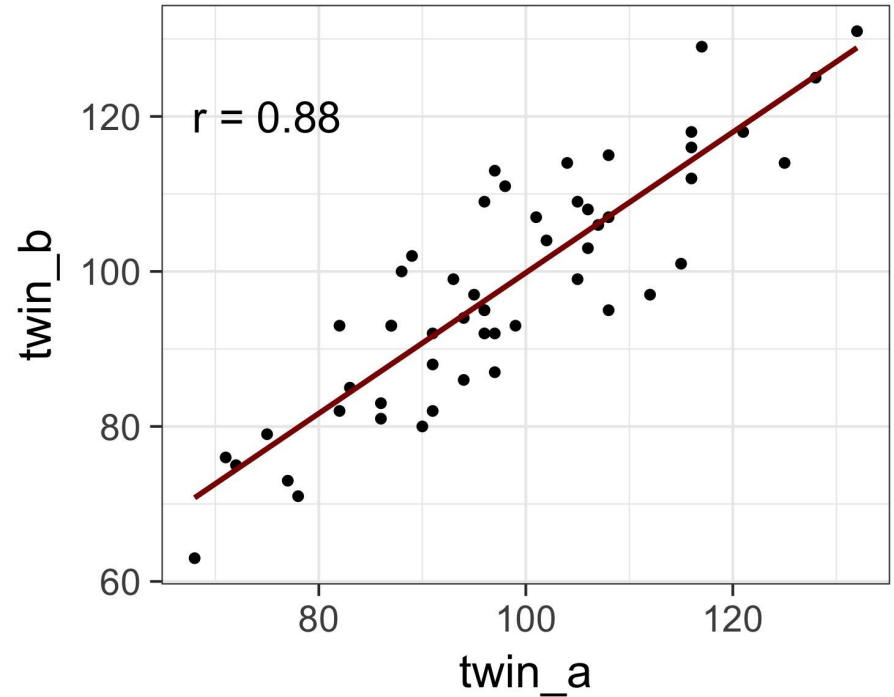
# Key ideas

1. A regression's slope codes the relationship between the two measures
2. Correlation is equivalent to the slope of a regression for standardized values
3. Inference for regression parameters uses t-tests

# Nature or nurture?

In 1966 Cyril Burt published a paper called "**The genetic determination of differences in intelligence: A study of monozygotic twins reared apart**"

The data consist of IQ scores for [an assumed random sample of] 53 identical twins, separated within 6-months of birth and raised apart



# Practice Question 1: Interpreting regression output

```
                Estimate   Std. Error t value Pr(>|t|)
(Intercept)    9.08670     6.92036   1.313   0.195
twin_a         0.90741     0.07004  12.957 <2e-16 ***
```

---

Residual standard error: 7.417 on 51 degrees of freedom

Multiple R-squared: 0.767, Adjusted R-squared: 0.7624

F-statistic: 167.9 on 1 and 51 DF, p-value: < 2.2e-16

**Which of the following is false?**

- (a) Additional 10 points in one twin's IQ is associated with additional 9 points in the the other twin's IQ, on average.
- (b) Roughly 91% of the variance in twins' IQs can be predicted by the model.
- (c) The linear model is  $\text{twin}_b = 9.08 + 0.91 \times \text{twin}_a$ .
- (d) Twins in group b with IQs higher than average IQs tend to have biological twins in group a with higher than average IQs as well.

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## Practice Question 2: Testing the relationship

Assuming that these 53 pairs of twins are a representative sample of all twins separated at birth, we would like to test if these data provide convincing evidence that the IQ of a biological twin is a significant predictor of IQ of the other twin.

**What are the appropriate hypotheses?**

- (a)  $H_0: b_0 = 0; H_A: b_0 \neq 0$
- (b)  $H_0: \beta_0 = 0; H_A: \beta_0 \neq 0$
- (c)  $H_0: b_1 = 0; H_A: b_1 \neq 0$
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# Analyzing the slope of the regression line

	estimate	std.error	t-value	p-value
(Intercept)	9.0867	6.9203	1.3130	0.1950
twin_a	0.9074	0.0700	12.956	0.0000

We always use a **t-test** in inference for regression.

Remember: test statistic  $T = (\text{point estimate} - \text{null value}) / SE$

Point estimate =  $b_1$  is the observed slope.  $SE_{b_1}$  is the standard error of the slope.

Degrees of freedom of the slope is  $df = n - 2$ , where  $n$  is the sample size.

Remember: we lose 1 degree of freedom for each parameter we estimate, and in simple linear regression we estimate 2 parameters,  $\beta_0$  and  $\beta_1$ .

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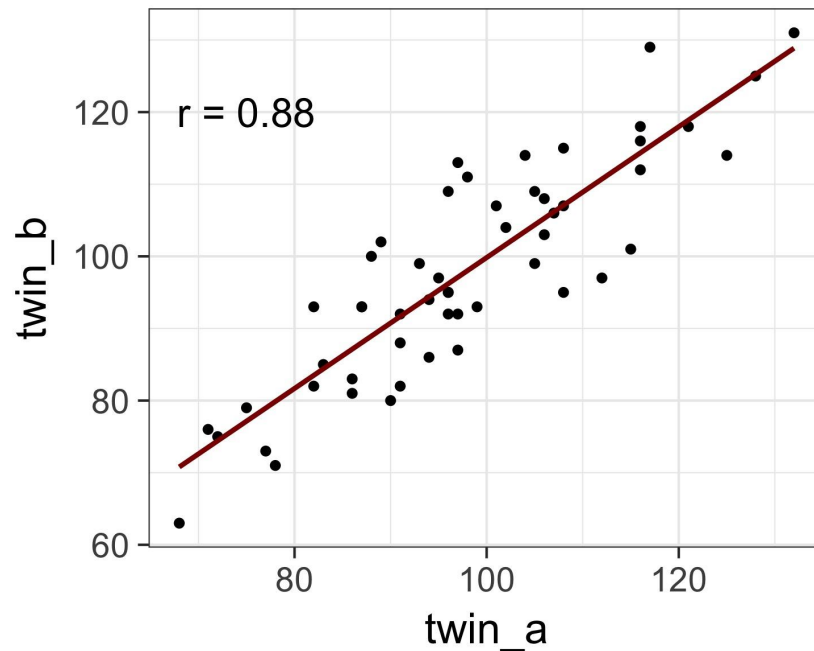
$$T = \frac{.9074 - 0}{.0700} = 12.956$$

$$df = 53 - 2 = 51$$

$$p - value = P(|T| > 12.956) < .001$$

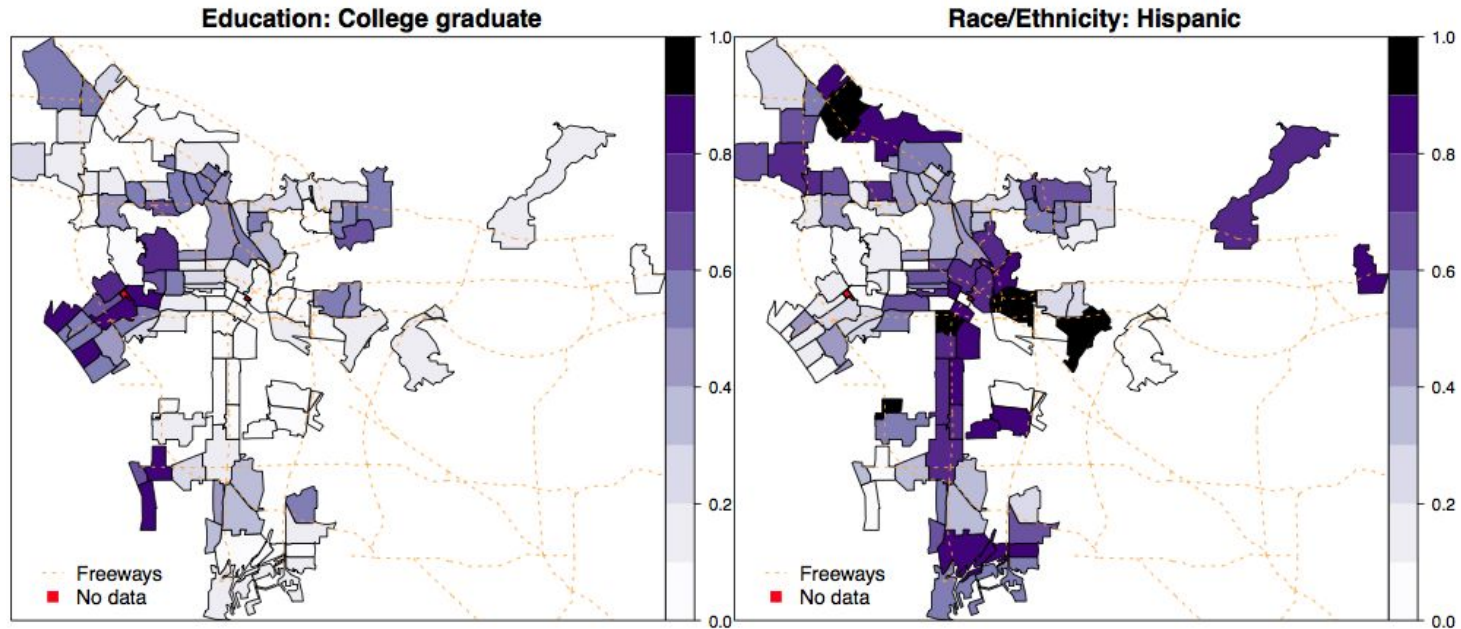
# What is the relationship between slope and correlation?

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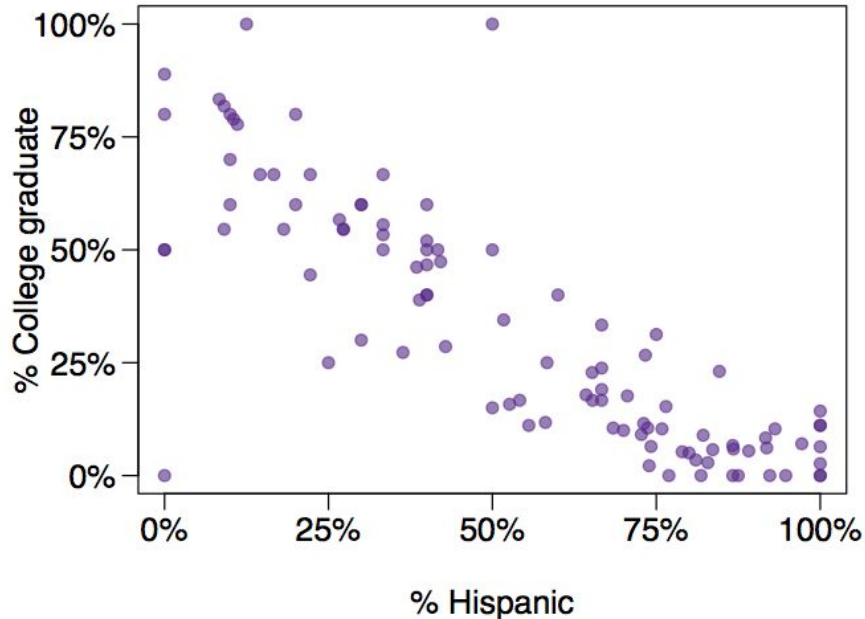
# Let's try another example

What can we say about the relationship between % college graduate and % Hispanic in a sample of 100 zip code areas in LA?



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## Practice Question 3: Inference from a logistic regression

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	0.7290	0.0308	23.68	0.0000
%Hispanic	-0.7527	0.0501	-15.01	0.0000

**Which of the following is the best interpretation of the slope?**

- (a) A 1% increase in Hispanic residents in a zip code area in LA is associated with a 75% decrease in % of college grads.
- (b) A 1% increase in Hispanic residents in a zip code area in LA is associated with a 0.75% decrease in % of college grads.
- (c) An additional 1% of Hispanic residents decreases the % of college graduates in a zip code area in LA by 0.75%.
- (d) In areas with no Hispanic residents, % college graduates is expected to be 75%.

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- (d) In areas with no Hispanic residents, % college graduates is expected to be 75%.



# So what should we conclude?

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	0.7290	0.0308	23.68	0.0000
%Hispanic	-0.7527	0.0501	-15.01	0.0000

Do these data provide convincing evidence for a significant relationship between % Hispanic and % college graduates in zip code areas in LA?

***Yes, the  $p$ -value for % Hispanic is low, indicating that the data provide convincing evidence that the slope parameter is different than 0.***

How reliable is this evidence if the zip codes are not randomly selected?

***Not very...***

## Practice Question 4: Confidence intervals for regression estimates

Remember that a confidence interval is calculated as point estimate  $\pm$  ME and the degrees of freedom associated with the slope in a simple linear regression is  $n - 2$ .

**Which of the below is the correct 95% confidence interval for the slope parameter? (Note that the model is based on observations from 53 twins).**

	estimate	std.error	t-value	p-value
(Intercept)	9.0867	6.9203	1.3130	0.1950
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- (a)  $9.0867 \pm 1.65 \times 6.9203$
- (b)  $.9074 \pm 2.01 \times .0700$
- (c)  $.9074 \pm 1.96 \times .0700$
- (d)  $9.0867 \pm 1.96 \times .0700$

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- (a)  $9.0867 \pm 1.65 \times 6.9203$        $n = 53$      $df = 53 - 2 = 51$
- (b)  $.9074 \pm 2.01 \times .0700$**        $95\%: t_{51}^* = 2.01$
- (c)  $.9074 \pm 1.96 \times .0700$        $0.9074 \pm 2.01 \times 0.0700$
- (d)  $9.0867 \pm 1.96 \times .0700$        $(0.767, 1.05)$

# Inference for linear regression

Inference for the slope for a single-predictor linear regression model:

$$\text{Hypothesis test: } T = \frac{b_1 - \text{null value}}{SE_{b_1}} \quad df = n - 2$$

$$\text{Confidence interval: } b_1 \pm t_{df=n-2}^* SE_{b_1}$$

The null value is often 0 since we are usually checking for **any** relationship between the explanatory and the response variable.

The regression output gives  $b_1$ ,  $SE_{b_1}$ , and **two-tailed** p-value for the t-test for the slope where the null value is 0.

We rarely do inference on the intercept, so we'll focusing on the slope.

# Key ideas

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2. Correlation is equivalent to the slope of a regression for standardized values
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